Question 1 (10p)

Explain the physics behind the formation of a river delta. (Relate your answer to all relevant physical principles we have discussed in the lectures of the course.)

Colloidal particles come with the fresh water. They carry charged groups on the surface which makes them repel each other and stabilizes the suspension so they do not sediment. By the sea the ionic strength of the water increases, which decreases the repulsion. Attractive forces (van der Waals) take over so the colloids aggregate. The aggregates are much heavier than the individual colloids so they sediment. As sediments are continually deposited on the bottom the water must find other paths and hence the delta is formed.

Question 2

A (5p)

Polyethylene ($-CH_2-CH_2-CH_2-$) chains have each been covalently bound to a hydrophilic chemical group. Assume each C-C bond is 1.5 Å and that each $-CH_2-$ unit has a volume of 3.3Å^3. How large (at least) must the area occupied by the hydrophilic group be for the molecules to form spherical micelles in water?

Assume there are N $-CH_2-$ units. The tail volume is then $3.3N$ Å^3 and the max tail length is $1.5N$ Å. The combination parameter $v/[lA]$ should be 1/3 at the limit of forming spheres. Thus $A = 3\times3.3N/[1.5N] = 6.6$ Å^2.

B (5p)

For this critical value of the "head group" area and a radius of the hydrophobic core equal to 3 nm, calculate the degree of polymerization for the polyethylene.

Assume there are M molecules in the micelle. The surface area is $4\pi R^2 = MA$, which you can solve for M (~1700). The hydrophobic volume is $4\pi R^3/3 = M\times3.3N$ which you can now solve for $N = 20$.

Question 3

A (5p)

The Alexander – de Gennes brush height is written as:

$$H = \left[\frac{\Gamma}{3}\right]^{\frac{1}{3}} a^{\frac{5}{3}} N$$

If a polymer brush consists of coils where each occupies an area on the surface equal to twice the square of the Kuhn length, what is the brush height expressed in percent of the contour length?

First rescale the expression so that H is proportional to aN. Replace Γ with $1/[2b^2]$. The Kuhn length will disappear from the expression and $H/[aN] = [1/6]^{1/3} = 55\%$.

B (5p)

The end to end distance *r* of an ideal chain in a theta solvent follows a Gaussian probability distribution (probability density function):

$$p(r,N)=\left[\frac{2\pi Na^2}{3}\right]^{-3/2}\exp\left(-\frac{3r^2}{2Na^2}\right)$$

Write an expression (you do not have to solve it) for the probability that a coil (at any point in time) has an end to end distance which is higher than half the contour length?

Hint: Note that the prefactor has the dimension of inverse volume.

This is a probability distribution in space which should be integrated with respect to the radial coordinate in spherical symmetry. If we use P for probability:

$dP = 4\pi r^2 p(r)dr$

The probability will be given by one minus the probability that the end to end distance is somewhere between zero and half the contour length (aN). Hence the expression is:

$$P\left(r>\frac{aN}{2}\right)=1-\int_0^{\frac{aN}{2}}4\pi r^2\left[\frac{2\pi Na^2}{3}\right]^{-3/2}\exp\left(-\frac{3r^2}{2Na^2}\right)dr$$